Differentiation- Mark Scheme

June 2019 Mathematics Advanced Paper 1: Pure Mathematics 1

1.

Question	Scheme	Marks	AOs
3 (a)	Correct method used in attempting to differentiate $y=\frac{5x^2+10x}{(x+1)^2}$	M1	3.1a
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x+1)^2\times(10x+10)-(5x^2+10x)\times2(x+1)}{(x+1)^4}$ oe	A1	1.1b
	Factorises/Cancels term in $(x+1)$ and attempts to simplify $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x+1)\times(10x+10)-(5x^2+10x)\times2}{(x+1)^3}=\frac{A}{(x+1)^3}$	M1	2.1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{10}{(x+1)^3}$	A1	1.1b
		(4)	
(b)	For $x<-1$ Follow through on their $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{A}{(x+1)^n}, n=1,3$	B1ft	2.2a
		(1)	
			(5 marks)

(a)

M1: Attempts to use a correct rule to differentiate Eg: Use of quotient (& chain) rules on $y=\frac{5x^2+10x}{(x+1)^2}$

Alternatively uses the product (and chain) rules on $y=(5x^2+10x)(x+1)^{-2}$

Condone slips but expect $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=\frac{(x+1)^2\times(Ax+B)-(5x^2+10x)\times(Cx+D)}{(x+1)^4}$ $(A,B,C,D>0)$ or

$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=\frac{(x+1)^2\times(Ax+B)-(5x^2+10x)\times(Cx+D)}{\left((x+1)^2\right)^2}$ $(A,B,C,D>0)$ using the quotient rule

or $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=(x+1)^{-2}\times(Ax+B)+(5x^2+10x)\times C(x+1)^{-3}$ $(A,B,C\neq0)$ using the product rule.

Condone missing brackets and slips for the M mark. For instance if they quote $u=5x^2+10$, $v=(x+1)^2$ and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow where they quote the correct formula, give values of u and v, but only have v rather than v^2 the denominator.

A1: A correct (unsimplified) answer

Eg. $\left(\frac{dy}{dx}\right)=\frac{(x+1)^2\times(10x+10)-(5x^2+10x)\times 2(x+1)}{(x+1)^4}$ or equivalent via the quotient rule.

OR $\left(\frac{dy}{dx}\right)=(x+1)^{-2}\times(10x+10)+(5x^2+10x)\times -2(x+1)^{-3}$ or equivalent via the product rule

M1: A valid attempt to proceed to the given form of the answer.

It is dependent upon having a quotient rule of $\pm\frac{v\mathrm{d}u-u\mathrm{d}v}{v^2}$ and proceeding to $\frac{A}{(x+1)^3}$

It can also be scored on a quotient rule of $\pm\frac{v\mathrm{d}u-u\mathrm{d}v}{v}$ and proceeding to $\frac{A}{(x+1)}$

You may see candidates expanding terms in the numerator. FYI $10x^3+30x^2+30x+10-10x^3-30x^2-20x$

but under this method they must reach the same expression as required by the main method.

Using the product rule expect to see a common denominator being used correctly before the above

A1: $\frac{dy}{dx}=\frac{10}{(x+1)^3}$ There is no requirement to see $\frac{dy}{dx}=$ and they can recover from missing brackets/slips.

(b)

B1ft: Score for deducing the correct answer of $x<-1$ This can be scored independent of their answer to part (a). Alternatively score for a correct **ft** answer for their $\frac{dy}{dx}=\frac{A}{(x+1)^n}$ where $A<0$ and $n=1,3$ award for $x>-1$. So for example if $A>0$ and $n=1,3\Rightarrow x<-1$

Question	Scheme	Marks	AOs
Alt via division	Writes $y=\frac{5x^2+10x}{(x+1)^2}$ in form $y=A\pm\frac{B}{(x+1)^2}$ $A,B\neq 0$	M1	3.1a
	Writes $y=\frac{5x^2+10x}{(x+1)^2}$ in the form $y=5-\frac{5}{(x+1)^2}$	A1	1.1b
	Uses the chain rule $\Rightarrow\frac{dy}{dx}=\frac{C}{(x+1)^3}$ (May be scored from $A=0$)	M1	2.1
	$\frac{dy}{dx}=\frac{10}{(x+1)^3}$ which cannot be awarded from incorrect value of A	A1	1.1b
		(4)	
(b)	For $x<-1$ or correct follow through	B1ft	2.2a
		(1)	
			(5 marks)

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2.

Question	Scheme	Marks	AOs
5	$\dfrac{dy}{d\theta}=\dfrac{(2\sin\theta+2\cos\theta)3\cos\theta-3\sin\theta(2\cos\theta-2\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$	M1 A1	1.1b 1.1b
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator or uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\dfrac{dy}{d\theta}=\dfrac{...}{.........C\sin\theta\cos\theta}$	M1	3.1a
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\dfrac{dy}{d\theta}=\dfrac{P}{Q+R\sin 2\theta}$	M1	2.1
	$\Rightarrow\dfrac{dy}{d\theta}=\dfrac{3}{2+2\sin 2\theta}=\dfrac{3/2}{1+\sin 2\theta}$	A1	1.1b
		(5 marks)	

Notes:

M1: For choosing either the quotient, product rule or implicit differentiation and applying it to the given function. Look for the correct form of $\dfrac{dy}{d\theta}$ (condone it being stated as $\dfrac{dy}{dx}$) but tolerate slips on the coefficients and also condone $\dfrac{d(\sin\theta)}{d\theta}=\pm\cos\theta$ and $\dfrac{d(\cos\theta)}{d\theta}=\pm\sin\theta$

For quotient rule look for $\dfrac{dy}{d\theta}=\dfrac{(2\sin\theta+2\cos\theta)\times\pm...\cos\theta-3\sin\theta(\pm...\cos\theta\pm...\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$

For product rule look for

$$\frac{dy}{d\theta}=(2\sin\theta+2\cos\theta)^{-1}\times\pm...\cos\theta\pm3\sin\theta\times(2\sin\theta+2\cos\theta)^{-2}\times(\pm...\cos\theta\pm...\sin\theta)$$

Implicit differentiation look for $(...\cos\theta\pm...\sin\theta)y+(2\sin\theta+2\cos\theta)\dfrac{dy}{d\theta}=...\cos\theta$

A1: A correct expression involving $\dfrac{dy}{d\theta}$ condoning it appearing as $\dfrac{dy}{dx}$

M1: Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator OR uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\dfrac{dy}{d\theta}=\dfrac{...}{.........C\sin\theta\cos\theta}$

M1: Expands and uses $\sin^2\theta+\cos^2\theta=1$ in the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in the denominator to reach an expression of the form $\dfrac{dy}{d\theta}=\dfrac{P}{Q+R\sin 2\theta}$.

A1: Fully correct proof with $A=\dfrac{3}{2}$ stated but allow for example $\dfrac{3/2}{1+\sin 2\theta}$

Allow recovery from missing brackets. Condone notation slips. This is not a given answer

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3.

Question	Scheme	Marks	AOs
5(a)	$x^n \to x^{n-1}$	M1	1.1b
	$\left(\frac{dy}{dx}\right) = 6x - \frac{24}{x^2}$	A1 A1	1.1b 1.1b
		(3)	
(b)	Attempts $6x - \frac{24}{x^2} > 0 \Rightarrow x >$	M1	1.1b
	$x > \sqrt[3]{4}$ or $x \geqslant \sqrt[3]{4}$	A1	2.5
		(2)	
			(5 marks)

(a)

M1: $x^n \to x^{n-1}$ for any correct index of x. Score for $x^2 \to x$ or $x^{-1} \to x^{-2}$

Allow for unprocessed indices. $x^2 \to x^{2-1}$ oe

A1: Sight of either $6x$ or $-\frac{24}{x^2}$ which may be un simplified.

Condone an additional term e.g. + 2 for this mark

The indices now must have been processed

A1: $\frac{dy}{dx} = 6x - \frac{24}{x^2}$ or exact simplified equivalent. Eg accept $\frac{dy}{dx} = 6x^1 - 24x^{-2}$

You do not need to see the $\frac{dy}{dx}$ and you should isw after a correct simplified answer.

(b)

M1: Sets an allowable $\frac{dy}{dx}$**...0** and proceeds to x... via an allowable intermediate equation

$\frac{dy}{dx}$ must be in the form $Ax + Bx^{-2}$ where $A, B \neq 0$

and the intermediate equation must be of the form $x^p ... q$ oe

Do not be concerned by either the processing, an equality or a different inequality.

It may be implied by $x =$ awrt 1.59

A1: $x > \sqrt[3]{4}$ or $x \geqslant \sqrt[3]{4}$ oe such as $x > 4^{\frac{1}{3}}$ or $x \geqslant 2^{\frac{2}{3}}$

4.

	Scheme	Marks	AOs
13.	The overall method of finding the x coordinate of A.	M1	3.1a
	$y=2x^3-17x^2+40x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=6x^2-34x+40$	B1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow 6x^2-34x+40=0 \Rightarrow 2(3x-5)(x-4)=0 \Rightarrow x=...$	M1	1.1b
	Chooses $x=4$ $\cancel{x=\frac{5}{3}}$	A1	3.2a
	$\int 2x^3-17x^2+40x\,\mathrm{d}x=\left[\frac{1}{2}x^4-\frac{17}{3}x^3+20x^2\right]$	B1	1.1b
	Area $=\frac{1}{2}(4)^4-\frac{17}{3}(4)^3+20(4)^2$	M1	1.1b
	$=\frac{256}{3}$ *	A1*	2.1
		(7)	
			(7 marks)

M1: An overall problem -solving method mark to find the minimum point. To score this you need to see

- an attempt to differentiate with at least **two correct terms**
- an attempt to set their $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and then solve to find x. Don't be overly concerned by the mechanics of this solution

B1: $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)6x^2-34x+40$ which may be unsimplified

M1: Sets their $\frac{\mathrm{d}y}{\mathrm{d}x}=0$, which must be a 3TQ in x, and attempts to solve via factorisation, formula or calculator. If a calculator is used to find the roots, they must be correct for their quadratic.
If $\frac{\mathrm{d}y}{\mathrm{d}x}$ is correct allow them to just choose the root 4 for M1 A1. Condone $(x-4)\left(x-\frac{5}{3}\right)$

A1: Chooses $x=4$ This may be awarded from the upper limit in their integral

B1: $\int 2x^3-17x^2+40x\,\mathrm{d}x=\left[\frac{1}{2}x^4-\frac{17}{3}x^3+20x^2\right]$ which may be unsimplified

M1: Correct attempt at area. There may be slips on the integration but expect **two correct terms**
The upper limit used must be their larger solution of $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and the lower limit used must be 0.
So if their roots are 6 and 10, then they must use 10 and 0. If only one value is found then the limits must be 0 to that value.
Expect to see embedded or calculated values.
Don't accept $\int_0^4 2x^3-17x^2+40x\,\mathrm{d}x=\frac{256}{3}$ without seeing the integration and the embedded or calculated values

A1*: Area $=\frac{256}{3}$ **with** correct notation and no errors. Note that this is a given answer.

For correct notation expect to see

- $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}}{\mathrm{d}x}$ used correctly at least once. If $\mathrm{f}(x)$ is used accept $\mathrm{f}'(x)$. Condone y'
- $\int 2x^3 - 17x^2 + 40x \, \mathrm{d}x$ used correctly at least once with or without the limits.

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5.

Question	Scheme	Marks	AOs
8 (a)(i)	$C = \frac{1500}{v} + \frac{2v}{11} + 60 \Rightarrow \frac{\mathrm{d}C}{\mathrm{d}v} = -\frac{1500}{v^2} + \frac{2}{11}$	M1 A1	3.1b 1.1b
	Sets $\frac{\mathrm{d}C}{\mathrm{d}v} = 0 \Rightarrow v^2 = 8250$	M1	1.1b
	$\Rightarrow v = \sqrt{8250} \Rightarrow v = 90.8\ (\mathrm{km\,h^{-1}})$	A1	1.1b
(ii)	For substituting their $v = 90.8$ in $C = \frac{1500}{v} + \frac{2v}{11} + 60$	M1	3.4
	Minimum cost =awrt (£) 93	A1 ft	1.1b
		(6)	
(b)	Finds $\frac{\mathrm{d}^2C}{\mathrm{d}v^2} = +\frac{3000}{v^3}$ at $v = 90.8$	M1	1.1b
	$\frac{\mathrm{d}^2C}{\mathrm{d}v^2} = (+0.004) > 0$ hence minimum (cost)	A1 ft	2.4
		(2)	
(c)	It would be impossible to drive at this speed over the whole journey	B1	3.5b
		(1)	
			(9 marks)

(a)(i)

M1: Attempts to differentiate (deals with the powers of v correctly).

Look for an expression for $\frac{dC}{dv}$ in the form $\frac{A}{v^2}+B$

A1: $\left(\frac{dC}{dv}\right)=-\frac{1500}{v^2}+\frac{2}{11}$

A number of students are solving part (a) numerically or graphically. Allow these students to pick up the M1 A1 here from part (b) when they attempt the second derivative.

M1: Sets $\frac{dC}{dv}=0$ (which may be implied) and proceeds to an equation of the type $v^n=k, k>0$

Allow here equations of the type $\frac{1}{v^n}=k, k>0$

A1: $v=\sqrt{8250}$ or $5\sqrt{330}$ awrt 90.8 (km h^{-1}) . Don't be concerned by incorrect / lack of units.

As this is a speed withhold this mark for answers such as $v=\pm\sqrt{8250}$

* Condone $\frac{dC}{dv}$ appearing as $\frac{dy}{dx}$ or perhaps not appearing at all. Just look for the rhs.

(a)(ii)

M1: For a correct method of finding $C=$ from their solution to $\frac{dC}{dv}=0$.

Do not accept attempts using negative values of v.
Award if you see $v=..,C=...$ where the v used is their solution to (a)(i). You do not need to check this calculation.
A1ft: Minimum cost = awrt (£) 93. Condone the omission of units

Follow through on sensible values of v. $60<v<110$

v	C
60	95.9
65	94.9
70	94.2
75	93.6
80	93.3
85	93.1
90	93.0
95	93.1
100	93.2
105	93.4
110	93.6

(b)

M1: Finds $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}$ (following through on their $\frac{\mathrm{d}C}{\mathrm{d}v}$ which must be of equivalent difficulty) and attempts to find its value / sign at their v

Allow a substitution of their answer to (a) (i) in their $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}$

Allow an explanation into the sign of $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}$ from its terms (as $v > 0$)

A1ft: $\frac{\mathrm{d}^2C}{\mathrm{d}v^2} = +0.004 > 0$ hence minimum (cost). Alternatively $\frac{\mathrm{d}^2C}{\mathrm{d}v^2} = +\frac{3000}{v^3} > 0$ as $v > 0$

Requires a correct calculation or expression, a correct statement and a correct conclusion.

Follow through on their v ($v > 0$) and their $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}$

* Condone $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}$ appearing as $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ or not appearing at all for the M1 but for the A1 the correct notation must be used (accept notation C'').

(c)

B1: Gives a limitation of the given model, for example

- It would be impossible to drive at this speed over the whole journey
- The traffic would mean that you cannot drive at a constant speed

Any statement that implies that the speed could not be constant is acceptable. Do not accept/ignore irrelevant statements such as "air resistance" etc

6.

Question	Scheme	Marks	AOs
10	Considers $\frac{(x+h)^3 - x^3}{h}$	B1	2.1
	Expands $(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$	M1	1.1b
	so gradient (of chord) $= \frac{3x^2h + 3xh^2 + h^3}{h} = 3x^2 + 3xh + h^2$	A1	1.1b
	States as $h \to 0$, $3x^2 + 3xh + h^2 \to 3x^2$ so derivative $= 3x^2$ *	A1*	2.5
			(4 marks)

Note: On e pen this is set up as B1 M1 M1 A1. We are scoring it B1 M1 A1 A1

B1: Gives the correct fraction for the gradient of the chord either $\dfrac{(x+h)^3-x^3}{h}$ or $\dfrac{(x+\delta x)^3-x^3}{\delta x}$

It may also be awarded for $\dfrac{(x+h)^3-x^3}{x+h-x}$ oe. It may be seen in an expanded form

It does not have to be linked to the gradient of the chord

M1: Attempts to expand $(x+h)^3$ or $(x+\delta x)^3$ Look for two correct terms, most likely $x^3+...+h^3$
This is independent of the B1

A1: Achieves gradient (of chord) is $3x^2+3xh+h^2$ or exact un simplified equivalent such as $3x^2+2xh+xh+h^2$. Again, there is no requirement to state that this expression is the gradient of the chord

A1*: CSO. Requires correct algebra and making a link between the gradient of the chord and the gradient of the curve. See below how the link can be made. The words "gradient of the chord" do not need to be mentioned but derivative, $\mathrm{f}'(x), \dfrac{\mathrm{d}y}{\mathrm{d}x}, y'$ should be. Condone invisible brackets for the expansion of $(x+h)^3$ as long as it is only seen at the side as intermediate working.

Requires either

- $\mathrm{f}'(x) \underset{\lim h\to 0}{=} \dfrac{(x+h)^3-x^3}{h} = 3x^2+3xh+h^2 = 3x^2$
- Gradient of chord $=3x^2+3xh+h^2$ As $h\to 0$ Gradient of chord tends to the gradient of curve so derivative is $3x^2$
- $\mathrm{f}'(x) \underset{\lim h\to 0}{=} 3x^2+3xh+h^2 = 3x^2$
- Gradient of **chord** $=3x^2+3xh+h^2$ when $h\to 0$ gradient of **curve** $=3x^2$
- Do not allow $h=0$ alone without limit being considered somewhere:
  so don't accept $h=0 \Rightarrow \mathrm{f}'(x)=3x^2+3x\times 0+0^2=3x^2$

....

Alternative: B1: Considers $\dfrac{(x+h)^3-(x-h)^3}{2h}$ M1: As above A1: $\dfrac{6x^2h^2+2h^3}{2h}=3x^2+h^2$

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7.

Question Number	Scheme		Marks
2.	$y=\sqrt{x}+\frac{4}{\sqrt{x}}+4=x^{\frac{1}{2}}+4x^{-\frac{1}{2}}+4$		
	$x^n \to x^{n-1}$	Decreases any power by 1. Either $x^{\frac{1}{2}} \to x^{-\frac{1}{2}}$ or $x^{-\frac{1}{2}} \to x^{-\frac{3}{2}}$ or $4 \to 0$ or $x^{\text{their}\,n} \to x^{\text{their}\,n-1}$ for fractional n.	M1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{1}{2}x^{-\frac{1}{2}}+4\times-\frac{1}{2}x^{-\frac{3}{2}}$ $\left(=\frac{1}{2}x^{-\frac{1}{2}}-2x^{-\frac{3}{2}}\right)$	Correct derivative, simplified or un-simplified including indices. E.g. allow $\frac{1}{2}-1$ for $-\frac{1}{2}$ and allow $-\frac{1}{2}-1$ for $-\frac{3}{2}$	A1
	$x=8 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}8^{-\frac{1}{2}}+4\times-\frac{1}{2}8^{-\frac{3}{2}}$	Attempts to substitute $x=8$ into their 'changed' (even integrated) expression that is clearly not y. If they attempt algebraic manipulation of their $\mathrm{d}y/\mathrm{d}x$ before substitution, this mark is still available.	M1
	$=\frac{1}{2\sqrt{8}}-\frac{2}{\left(\sqrt{8}\right)^3}=\frac{1}{2\sqrt{8}}-\frac{2}{8\sqrt{8}}=\frac{1}{8\sqrt{2}}=\frac{1}{16}\sqrt{2}$	B1: $\sqrt{8}=2\sqrt{2}$ seen or implied anywhere, including from substituting $x=8$ into y. May be seen explicitly or implied from e.g. $8^{\frac{3}{2}}=16\sqrt{2}$ or $8^{\frac{5}{2}}=128\sqrt{2}$ or $4\sqrt{8}=8\sqrt{2}$ A1: cso $\frac{1}{16}\sqrt{2}$ or $\frac{\sqrt{2}}{16}$ and allow rational equivalents for $\frac{1}{16}$ e.g. $\frac{32}{512}$ Apply isw so award this mark as soon as a correct answer is seen.	B1A1
			(5 marks)

8.

Question Number	Scheme		Marks
7.(a)	$f'(4) = 30 + \dfrac{6-5\times4^2}{\sqrt{4}}$	Attempts to substitutes $x = 4$ into $f'(x) = 30 + \dfrac{6-5x^2}{\sqrt{x}}$ or their algebraically manipulated $f'(x)$	M1
	$f'(4) = -7$	Gradient = -7	A1
	$y-(-8) = "-7"\times(x-4)$ or $y = "-7"x + c \Rightarrow -8 = "-7"\times 4 + c \Rightarrow c = ...$	Attempts an equation of a tangent using their numeric $f'(4)$ which has come from substituting $x = 4$ into the given $f'(x)$ or their algebraically manipulated $f'(x)$ and $(4,-8)$ with the 4 and -8 correctly placed. If using $y = mx + c$, must reach as far as $c = \ldots$	M1
	$y = -7x + 20$	Cao. Allow $y = 20 - 7x$ and allow the "y =" to become "detached" but it must be present at some stage. E.g. $y = ...$ $= -7x + 20$	A1
			(4)
(b)	**Allow the marks in (b) to score in (a) i.e. <u>mark (a) and (b) together</u>**		
	$\Rightarrow f(x) = 30x + 6\dfrac{x^{\frac{1}{2}}}{0.5} - 5\dfrac{x^{\frac{5}{2}}}{2.5}(+c)$	M1: $30 \to 30x$ or $\dfrac{6}{\sqrt{x}} \to \alpha x^{\frac{1}{2}}$ or $-\dfrac{5x^2}{\sqrt{x}} \to \beta x^{\frac{5}{2}}$ (these cases only) A1: Any 2 correct terms which can be simplified or un-simplified. This includes the powers – so allow $-\frac{1}{2}+1$ for $\frac{1}{2}$ and allow $\frac{3}{2}+1$ for $\frac{5}{2}$ (With or without + c) A1: All 3 terms correct which can be simplified or un-simplified. (With or without + c)	M1A1A1
	Ignore any spurious integral signs		
	$x = 4, f(x) = -8 \Rightarrow$ $-8 = 120 + 24 - 64 + c \Rightarrow c = ...$	Substitutes $x = 4, f(x) = -8$ into their f(x) (not $f'(x)$) i.e. a changed $f'(x)$ containing $+c$ and rearranges to obtain a value or numerical expression for c.	M1
	$\Rightarrow (f(x) =) 30x + 12x^{\frac{1}{2}} - 2x^{\frac{5}{2}} - 88$	Cao and cso (Allow $\sqrt{x}$ for $x^{\frac{1}{2}}$ and e.g. $\sqrt{x^5}$ or $x^2\sqrt{x}$ for $x^{\frac{5}{2}}$). Isw here so as soon as you see the correct answer, award this mark. Note that the "f(x) =" is not needed.	A1
			(5)
			(9 marks)

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9.

Question Number	Scheme	Notes	Marks
7.	$y=3x^2+6x^{\frac{1}{3}}+\dfrac{2x^3-7}{3\sqrt{x}}$		
	$\dfrac{2x^3-7}{3\sqrt{x}}=\dfrac{2x^3}{3\sqrt{x}}-\dfrac{7}{3\sqrt{x}}=\dfrac{2}{3}x^{\frac{5}{2}}-\dfrac{7}{3}x^{-\frac{1}{2}}$	Attempts to split the fraction into 2 terms and obtains either $\alpha x^{\frac{5}{2}}$ or $\beta x^{-\frac{1}{2}}$. This may be implied by a correct power of x in their differentiation of one of these terms. But beware of $\beta x^{-\frac{1}{2}}$ coming from $\dfrac{2x^3-7}{3\sqrt{x}}=2x^3-7+3x^{-\frac{1}{2}}$	M1
	$x^n \to x^{n-1}$	Differentiates by reducing power by one for any of their powers of x	M1
	$\left(\dfrac{dy}{dx}=\right)6x+2x^{-\frac{2}{3}}+\dfrac{5}{3}x^{\frac{3}{2}}+\dfrac{7}{6}x^{-\frac{3}{2}}$	A1: $6x$. Do not accept $6x^1$. Depends on second M mark only. Award when first seen and isw. A1: $2x^{-\frac{2}{3}}$. Must be simplified so do not accept e.g. $\dfrac{2}{1}x^{-\frac{2}{3}}$ but allow $\dfrac{2}{\sqrt[3]{x^2}}$. Depends on second M mark only. Award when first seen and isw. A1: $\dfrac{5}{3}x^{\frac{3}{2}}$. Must be simplified but allow e.g. $1\dfrac{2}{3}x^{1.5}$ or e.g. $\dfrac{5}{3}\sqrt{x^3}$. Award when first seen and isw. A1: $\dfrac{7}{6}x^{-\frac{3}{2}}$. Must be simplified but allow e.g. $1\dfrac{1}{6}x^{-1\frac{1}{2}}$ or e.g. $\dfrac{7}{6\sqrt{x^3}}$. Award when first seen and isw.	A1A1A1A1
	In an otherwise fully correct solution, penalise the presence of + c by deducting the final A1		
			[6]

10.

Question Number	Scheme		Marks
11. (a)	$y=2x^3+kx^2+5x+6$		
	$\left(\dfrac{dy}{dx}=\right)6x^2+2kx+5$	M1: $x^n \to x^{n-1}$ for one of the terms including $6 \to 0$ A1: Correct derivative	M1 A1
			[2]

(b)	Gradient of given line is $\frac{17}{2}$	Uses or states $\frac{17}{2}$ or equivalent e.g. 8.5. Must be stated or used in (b) and not just seen as part of $y=\frac{17}{2}x+\frac{1}{2}$.	B1
	$\left(\frac{dy}{dx}\right)_{x=-2}=6(-2)^2+2k(-2)+5$	Substitutes $x=-2$ into their derivative **(not the curve)**	M1
	$"24-4k+5"="\frac{17}{2}"\Rightarrow k=\frac{41}{8}$	dM1: Puts their expression = their $\frac{17}{2}$ (Allow BOD for 17 or -17 but **not** the normal gradient) and solves to obtain a value for k. **Dependent on the previous method mark**. A1: $\frac{41}{8}$ or $5\frac{1}{8}$ or 5.125	dM1 A1
	Note: $6x^2+2kx+5=\frac{17}{2}x+\frac{1}{2}$ scores no marks on its own but may score the first M mark if they substitute x = -2 into the lhs. If they rearrange this equation and then substitute x = -2, this scores no marks.		
			[4]
(c)	$y=-16+4k-10+6=4"k"-20=\frac{1}{2}$	M1: Substitutes $x=-2$ and their numerical k into $y=\ldots$ A1: $y=\frac{1}{2}$	M1 A1
	Allow the marks for part (c) to be scored in part (b).		
			[2]
(d)	$y-"\frac{1}{2}"="\frac{17}{2}"(x-{}^-2)\Rightarrow -17x+2y-35=0$ or $y="\frac{17}{2}"x+c\Rightarrow c=\ldots\Rightarrow -17x+2y-35=0$ or $2y-17x=1+34\Rightarrow -17x+2y-35=0$	M1: Correct attempt at linear equation with their 8.5 gradient (not the normal gradient) using x = -2 and their $\frac{1}{2}$ A1: cao (allow any integer multiple)	M1 A1
			[2]
			10 marks

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11.

Question Number	Scheme		Marks
3.	$y=4x^3-\frac{5}{x^2}$		
(a)	$12x^2+\frac{10}{x^3}$	M1: $x^n\to x^{n-1}$ e.g. Sight of x^2 or x^{-3} or $\frac{1}{x^3}$ A1: $3\times4x^2$ or $-5\times-2x^{-3}$(oe) (Ignore + c for this mark) A1: $12x^2+\frac{10}{x^3}$ or $12x^2+10x^{-3}$ <u>all on one line</u> and no + c	M1A1A1

	Apply ISW here and award marks when first seen.		
			(3)
(b)	$x^4+\frac{5}{x}+c$ or $x^4+5x^{-1}+c$	M1: $x^n \to x^{n+1}$. e.g. Sight of x^4 or x^{-1} or $\frac{1}{x^1}$ **Do not award for integrating their answer to part (a)**	M1A1A1
		A1: $4\frac{x^4}{4}$ or $-5\times\frac{x^{-1}}{-1}$	
		A1: For fully correct and simplified answer with + c all on one line. Allow $x^4+5\times\frac{1}{x}+c$ Allow $1x^4$ for x^4	
	Apply ISW here and award marks when first seen. Ignore spurious integral signs for all marks.		
			(3)
			(6 marks)

12.

Question Number	Scheme		Marks
6(a)	$(x^2+4)(x-3)=x^3-3x^2+4x-12$	Attempt to multiply out the numerator to get a cubic with 4 terms and at least 2 correct	M1
	$\frac{x^3-3x^2+4x-12}{2x}=\frac{x^2}{2}-\frac{3}{2}x+2-6x^{-1}$	M1: Attempt to divide each term by $2x$. The powers of x of at least two terms must follow from their expansion. Allow an attempt to multiply by $2x^{-1}$	M1A1
		A1: Correct expression. May be un-simplified but powers of x must be combined e.g. $\frac{x^2}{2}$ not $\frac{x^3}{2x}$	
	$\frac{dy}{dx}=x-\frac{3}{2}+\frac{6}{x^2}$ oe e.g. $\frac{2x^3-3x^2+12}{2x^2}$	ddM1: $x^n \to x^{n-1}$ or $2\to 0$ **Dependent on both previous method marks.**	ddM1A1
		A1: $x-\frac{3}{2}+\frac{6}{x^2}$ oe and **isw** Accept $1x$ or even $1x^1$ but not $\frac{2x}{2}$ and not x^0. If they lose the previous A1 because of an **incorrect constant only** then allow recovery here and in part (b) for a correct derivative.	
			(5)

(b)	At $x=-1,\ y=10$	Correct value for y	B1
	$\left(\frac{dy}{dx}=\right)-1-\frac{3}{2}+\frac{6}{1}=3.5$	M1: Substitutes $x=-1$ into their expression for dy/dx A1: 3.5 oe cso	M1A1
	$y-'10'='3.5'(x--1)$	Uses their **tangent** gradient **which must come from calculus** with $x=-1$ and their numerical y with a correct straight line method. If using $y=mx+c$, this mark is awarded for correctly establishing a value for c.	M1
	$2y-7x-27=0$	$\pm k(2y-7x-27)=0$ cso	A1
			(5)
			(10 marks)

13.

Question Number	Scheme		Marks
10(a)	$f(x)=x^{\frac{3}{2}}-\frac{9}{2}x^{\frac{1}{2}}+2x(+c)$	M1: $x^n \to x^{n+1}$ A1: Two terms in x correct, simplification is not required in coefficients or powers A1: All terms in x correct. Simplification not required in coefficients or powers and + c is not required	M1A1A1
	Sub $x=4, y=9$ into $f(x) \Rightarrow c=...$	M1: Sub $x=4$, $y=9$ into f (x) to obtain a value for c. If no + c then M0. Use of $x=9$, $y=4$ is M0.	M1
	$(f(x)=)x^{\frac{3}{2}}-\frac{9}{2}x^{\frac{1}{2}}+2x+2$	Accept equivalents **but must be simplified** e.g. $f(x)=x^{\frac{3}{2}}-4.5\sqrt{x}+2x+2$ Must be all 'on one line' **and simplified**. Allow $x\sqrt{x}$ for $x^{\frac{3}{2}}$	A1
			(5)
(b)	Gradient of normal is $-\frac{1}{2} \Rightarrow$ Gradient of tangent $=+2$	M1: Gradient of $2y+x=0$ is $\pm\frac{1}{2}(m) \Rightarrow \frac{dy}{dx}=-\frac{1}{\pm\frac{1}{2}}$ A1: Gradient of tangent = +2 (May be implied)	M1A1
	The A1 may be implied by $\frac{-1}{\frac{3\sqrt{x}}{2}-\frac{9}{4\sqrt{x}}+2}=-\frac{1}{2}$		

	$\frac{3\sqrt{x}}{2}-\frac{9}{4\sqrt{x}}+2=2\Rightarrow\frac{3\sqrt{x}}{2}-\frac{9}{4\sqrt{x}}=0$	Sets the given $f'(x)$ **or their** $f'(x)$ $=$ their **changed** m and **not** their m where m has come from $2y+x=0$	M1
	$\times 4\sqrt{x}\Rightarrow 6x-9=0\Rightarrow x=..$	$\times 4\sqrt{x}$ or equivalent **correct algebraic** processing (**allow sign/arithmetic errors only**) and attempt to solve to obtain a value for x. If $f'(x)\neq 2$ they need to be solving a three term quadratic in $\sqrt{x}$ correctly and square to obtain a value for x. **Must be using the given** $f'(x)$ **for this mark.**	M1
	$x=1.5$	$x=\frac{3}{2}$ (1.5) Accept equivalents e.g . $x=\frac{9}{6}$ **If any 'extra' values are not rejected, score A0.**	A1
			(5)
	Beware $\frac{-1}{\frac{3\sqrt{x}}{2}-\frac{9}{4\sqrt{x}}+2}=-\frac{1}{2}\Rightarrow\frac{-2}{3\sqrt{x}}+\frac{4\sqrt{x}}{9}-\frac{1}{2}=-\frac{1}{2}$ etc. leads to the correct answer and could score M1A1M1M0(incorrect processing)A0		
			(10 marks)

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14.

Question Number	Scheme	Marks
7.	(a) $(1-2x)^2=1-4x+4x^2$ $\frac{d}{dx}(1-2x)^2=\frac{d}{dx}(1-4x+4x^2)=-4+8x$ o.e.	M1 M1A1 (3)
	Alternative method using chain rule: Answer of -4 (1 – 2 x)	M1M1A1 (3)
	(b) $\frac{x^5+6\sqrt{x}}{2x^2}=\frac{x^5}{2x^2}+6\frac{\sqrt{x}}{2x^2},=\frac{1}{2}x^3+3x^{-\frac{3}{2}}$ Attempts to differentiate $x^{-\frac{3}{2}}$ to give $k\ x^{-\frac{5}{2}}$ $=\frac{3}{2}x^2-\frac{9}{2}x^{-\frac{5}{2}}$ o.e. Quotient Rule (May rarely appear) – See note below	M1,A1 M1 A1 (4) **(7 marks)**

Notes

(a) M1 Attempt to multiply out bracket. Must be 3 or 4 term quadratic and **must have constant term 1**

M1 $x^n\to x^{n-1}$. Follow through on any term in an incorrect expression. Accept a constant $\to 0$

A1 $-4+8x$ Accept -4 (1 – 2x) or equivalent. This is not cso and may follow error in the constant term Following correct answer by -2 + 4x – apply isw

Correct answer with no working – assume chain rule and give M1M1A1 i.e. 3/3

Common errors: $(1-2x)^2=2-4x+4x^2$ is M0, then allow M1A1 for -4 + 8x

$(1-2x)^2=1-4x^2$ is M0 then -8x earns M1A0 or $(1-2x)^2=1-2x^2$ is M0 then -4x earns M1A0

15.

Question Number	Scheme	Marks
11.	(a) Discriminant = $b^2-4ac=8^2-4\times2\times3,=40$	M1, A1 (2)
	(b) $2x^2+8x+3=2(x^2+\text{...........})$ or p=2	B1
	$=2((x+2)^2\pm...)$ or $q=2$	M1
	$=2(x+2)^2-5$ or $p=2,\ q=2$ and $r=$ -5	A1 (3)
	(c) Method 1A: Sets derivative $"4x+8"=4\Rightarrow\ x=,\quad x=-1$	M1, A1
	Substitute $x=-1$ in $y=2x^2+8x+3\quad(\Rightarrow y=-3)$	dM1
	Substitute $x=-1$ and $y=-3$ in $y=4x+c$ or into $(y+3)=4(x+1)$ and expand	dM1
	$c=1$ or writing $y=4x+1$	A1cso (5)
	Method 1B: Sets derivative $"4x+8"=4\Rightarrow\ x=,\quad x=-1$	M1, A1
	Substitute $x=-1$ in $2x^2+8x+3\ =\ 4x+c$	dM1
	Attempts to find value of c	dM1
	$c=1$ or writing $y=4x+1$	A1cso (5)
	Method 2: Sets $2x^2+8x+3=4x+c$ and collects x terms together	M1
	Obtains $2x^2+4x+3-c=0$ or equivalent	A1
	States that $b^2-4ac=0$	dM1
	$4^2-4\times2\times(3-c)=0$ and so $c=$	dM1
	$c=1$	A1cso (5)
	Method 3: Sets $2x^2+8x+3=4x+c$ and collects x terms together	M1
	Obtains $2x^2+4x+3-c=0$ or equivalent	A1
	Uses $2(x+1)^2-2+3-c=0$ or equivalent	dM1
	Writes $-2+3-c=0$	dM1
	So $c=1$	A1cso (5)
	Also see special case for using a perpendicular gradient (overleaf)	**(10 marks)**

Notes

(a) M1 Attempts to calculate b^2-4ac using $8^2-4\times2\times3$ - must be correct – not just part of a quadratic formula
A1 Cao 40

(b) B1 See 2(….) or $p=2$
M1 $..((x+2)^2\pm...)$ is sufficient evidence or obtaining $q=2$
A1 Fully correct values. $2(x+2)^2-5$ or $p=2$, $q=2$, $r=-5$ cso.
Ignore inclusion of "=0".

[In many respects these marks are similar to three B marks.
$p=2$ is B1; $q=2$ is B1 and $p=2,\ q=2$ and $r=$ -5 is final B1 but they must be entered on epen as **B1 M1 A1**]

Special case: Obtains $2x^2+8x+3=2(x+2)-1$ This may have first B1, for $p=2$ then M0A0 (c) Method 1A (Differentiates and puts gradient equal to 4. Needs both x and y to find c)

M1 Attempts to solve their $\frac{dy}{dx} = 4$. They must reach $x = ...$ (Just differentiating is M0 A0)

A1 $x = -1$ (If this follows $\frac{dy}{dx} = 4x + 8$, then give M1 A1 by implication)

dM1 (Depends on previous M mark) Substitutes **their** $x = -1$ into f(x) or into "their f(x) from (b)" to find y

dM1 (Depends on both previous M marks) Substitutes **their** $x = -1$ and **their** $y = -3$ values into $y = 4x + c$ to find c or uses equation of line is $(y + \text{"3"}) = 4(x + \text{"1"})$ and rearranges to $y = mx + c$

A1 $c = 1$ or allow for $y = 4x + 1$ cso

(c) Method 1B (Differentiates and puts gradient equal to 4. Also equates equations and uses x to find c)

M1A1 Exactly as in Method 1A above

dM1 (Depends on previous M mark) Substitutes **their** $x = -1$ into $2x^2 + 8x + 3 = 4x + c$

dM1 Attempts to find value of c then A1 as before

(c) Method 2 (uses repeated root to find c by discriminant)

M1 Sets $2x^2 + 8x + 3 = 4x + c$ and tries to collect x terms together

A1 Collects terms e.g. $2x^2 + 4x + 3 - c = 0$ or $-2x^2 - 4x - 3 + c = 0$ or $2x^2 + 4x + 3 = c$ or even $2x^2 + 4x = c - 3$ Allow "=0" to be missing on RHS.

dM1 (If the line is a tangent it meets the curve at just one point so repeated root and $b^2 - 4ac = 0$) Stating that $b^2 - 4ac = 0$ is enough

dM1 Using $b^2 - 4ac = 0$ to obtain equation in terms of c (Eg. $4^2 - 4 \times 2 \times (3 - c) = 0$) AND leading to a solution for c

A1 $c = 1$ or allow for $y = 4x + 1$ cso

(c) Method 3 (Similar to method 2 but uses completion of the square on the quadratic to find repeated root)

M1 Sets $2x^2 + 8x + 3 = 4x + c$ and tries to collect x terms together. May be implied by $2x^2 + 8x + 3 - 4x \pm c$ on one side

A1 Collects terms e.g. $2x^2 + 4x + 3 - c = 0$ or $-2x^2 - 4x - 3 + c = 0$ or $2x^2 + 4x + 3 = c$ or even $2x^2 + 4x = c - 3$ Allow "=0" to be missing on RHS.

dM1 Then use completion of square $2(x + 1)^2 - 2 + 3 - c = 0$ (Allow $2(x + 1)^2 - k + 3 - c = 0$) where k is non zero. It is enough to give the correct or almost correct (with k) completion of the square

dM1 $-2 + 3 - c = 0$ AND leading to a solution for c (Allow $-1 + 3 - c = 0$) ($x = -1$ has been used)

A1 $c = 1$ cso

In Method 1 they may use part (b) and differentiate their f(x) and put it equal to 4
They can earn M1, but do not follow through errors.

In Methods 2 and 3 they may use part (b) to write
their $2(x + 2)^2 - 5 = 4x + c$. They need to expand and collect x terms together for M1
Then expanding gives $2x^2 + 4x + 3 - c = 0$ for A1 – do not follow through errors
Then the scheme is as before

If they just state $c = 1$ with little or no working – please send to review,

PTO for special case

Special case uses perpendicular gradient (maximum of 2/5)

Sets	$4x + 8 = -\frac{1}{4} \Rightarrow x = ,\quad x = -\frac{33}{16}$	M1 A0
Substitute	$x = -\frac{33}{16}$ in $y = 2x^2 + 8x + 3$ $\left(\Rightarrow y = -\frac{639}{128}\right)$	M0
Substitute	$x = -\frac{33}{16}$ and $y = -\frac{639}{128}$ into $y = 4x + c$ or into $(y + \frac{639}{128}) = 4(x + \frac{33}{16})$ and expand	M1 A0

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16.

Question Number	Scheme		Marks
9 (a)	$(3-x^2)^2 = 9-6x^2+x^4$	An attempt to expand the numerator obtaining an expression of the form $9 \pm px^2 \pm qx^4, \quad p,q \neq 0$	M1
	$9x^{-2}+x^2$	Must come from $\frac{9+x^4}{x^2}$	A1
	-6	Must come from $\frac{-6x^2}{x^2}$	A1
	Alternative 1: Writes $\frac{(3-x^2)^2}{x^2}$ as $(3x^{-1}-x)^2$ and attempts to expand = M1 then A1A1 as in the scheme.		
	Alternative 2: Sets $(3-x^2)^2 = 9+Ax^2+Bx^4$, expands $(3-x^2)^2$ and compares coefficients = M1 then A1A1 as in the scheme.		
			(3)
	$(\mathrm{f}'(x) = 9x^{-2}-6+x^2)$		
(b)	$-18x^{-3}+2x$	M1: $x^n \to x^{n-1}$ on separate terms at least once. Do not award for $A \to 0$ (Integrating is M0) A1ft: $-18x^{-3}+2$"B"x with a numerical B and no extra terms. (A may have been incorrect or even zero)	M1 A1ft
			(2)

(c)	$\mathrm{f}(x) = -9x^{-1} - 6x + \frac{x^3}{3}(+c)$	M1: $x^n \to x^{n+1}$ on separate terms at least once. (Differentiating is M0) A1ft: $-9x^{-1} + Ax + \frac{Bx^3}{3}(+c)$ with numerical A and B, $A, B \neq 0$	M1A1ft
	$10 = \frac{-9}{-3} - 6(-3) + \frac{(-3)^3}{3} + c$ so $c = \ldots$	Uses $x = -3$ and $y = 10$ in what they think is f(x) (They may have differentiated here) but it must be a changed function i.e. not the original $\mathrm{f}'(x)$, to form a linear equation in c and attempts to find c. No $+ c$ gets M0 and A0 unless their method implies that they are correctly finding a constant.	M1
	$c = -2$	cso	A1
	$(\mathrm{f}(x) =) - 9x^{-1} - 6x + \frac{x^3}{3} +$ their c	Follow through their c in an otherwise (possibly un-simplified) **correct expression.** Allow $-\frac{9}{x}$ for $-9x^{-1}$ or even $\frac{9x^{-1}}{-1}$.	A1ft
	Note that if they integrate in (b), no marks there but if they then go on to use their integration in (c), the marks for integration are available.		
			(5)
			[10]

17.

Question Number	Scheme		Marks
11 (a)	$\left(-\frac{3}{4}, 0\right)$. Accept $x = -\frac{3}{4}$		B1
			(1)
(b)	$y = 4$	B1: One correct asymptote	B1B1
	$x = 0$ or 'y-axis'	B1: Both correct asymptotes and no extra ones.	
	Special case $x \neq 0$ **and** $y \neq 4$ scores B1B0		
			(2)
(c)	$\frac{\mathrm{d}y}{\mathrm{d}x} = -3x^{-2}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = kx^{-2}$ (Allow $\frac{\mathrm{d}y}{\mathrm{d}x} = kx^{-2} + 4$)	M1
	At $x = -3$, gradient of curve $= -\frac{1}{3}$	Cao (may be un-simplified but must be a fraction with no powers) e.g. $-3(-3)^{-2}$ scores A0 unless evaluated as e.g. $\frac{-3}{9}$ or is implied by their normal gradient.	A1

	Gradient of normal = -1/*m*	Correct perpendicular gradient rule applied to a numerical gradient that must have come from substituting x = -3 into their derivative. **Dependent on the previous M1.**	dM1
	Normal at *P* is $(y-3)=3(x+3)$	M1: Correct straight line method using (-3, 3) and a "changed" gradient. A wrong equation with no formula quoted is M0. **Also dependent on the first M1.** A1: Any correct equation	dM1A1
			(5)
(d)	(-4, 0) and (0, 12).	Both correct (May be seen on a sketch)	B1
	So *AB* has length $\sqrt{160}$ or AB^2 has length 160	M1: Correct use of Pythagoras for their *A* and *B* one of which lies on the x-axis and the other on the y-axis, obtained from their equation in (c). A correct method for AB^2 or *AB*. A1: $\sqrt{160}$ or better e.g. $4\sqrt{10}$ **with no errors seen**	M1 A1cso
			(3)
			[11]

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18.

Question Number	Scheme		Marks
11.	$C: y = 2x - 8\sqrt{x} + 5, \quad x \geqslant 0$		
(a)	So, $y = 2x - 8x^{\frac{1}{2}} + 5$		
	$\frac{dy}{dx} = 2 - 4x^{-\frac{1}{2}} + \{0\} \qquad (x > 0)$		M1 A1 A1
			[3]
(b)	(When $x = \frac{1}{4}$, $y = 2(\frac{1}{4}) - 8\sqrt{(\frac{1}{4})} + 5$ so) $\quad y = \frac{3}{2}$		B1
	(gradient $= \frac{dy}{dx} =$) $2 - \frac{4}{\sqrt{(\frac{1}{4})}}$ $\{= -6\}$		M1
	Either: $y - "\frac{3}{2}" = "-6"(x - \frac{1}{4})$	**or**: $y = "-6"x + c$ and $"\frac{3}{2}" = "-6"(\frac{1}{4}) + c \Rightarrow c = "3"$	dM1
	So $\underline{y = -6x + 3}$		A1
			[4]
(c)	Tangent at Q is parallel to $2x - 3y + 18 = 0$		
	($y = \frac{2}{3}x + 6 \Rightarrow$) Gradient $= \frac{2}{3}$. so tangent gradient is $\frac{2}{3}$		B1
	So, $"2 - \frac{4}{\sqrt{x}}" = "\frac{2}{3}"$	Sets their gradient function = their numerical gradient.	M1
	$\Rightarrow \frac{4}{3} = \frac{4}{\sqrt{x}} \Rightarrow x = 9$	Ignore extra answer $x = -9$	A1
	When $x = 9$, $y = 2(9) - 8\sqrt{9} + 5 = -1$	Substitutes their found x into equation of curve.	dM1
		$y = -1$.	A1
			[5]
			12 marks

	Notes
(a)	**M1:** Evidence of differentiation, so $x^n \to x^{n-1}$ at least once so $x^1 \to 1$ or x^0 or $x^{\frac{1}{2}} \to x^{-\frac{1}{2}}$ not just $5 \to 0$
	A1: Any two of the three terms correct – do not need to see zero – the 5 disappearing is sufficient; need not be simplified.
	A1: $2 - 4x^{-\frac{1}{2}}$ Both terms correct, and simplified. Do not need to include domain $x > 0$
(b)	**B1:** Obtaining $y = 3/2$ or fractional or decimal equivalent (no working need be seen)
	M1: An attempt to substitute $x = \frac{1}{4}$ into $\frac{dy}{dx}$ to establish **gradient**. This may be implied by -6 or $m = -6$ but not $y = -6$. Can earn this M mark if they go on to use $m = \frac{1}{6}$ or use their numerical value of $\frac{dy}{dx}$.
	dM1: This depends on previous method mark. Complete method for obtaining the equation of the tangent, using their **tangent** gradient and their value for y_1 (obtained from $x = ¼$, allow slip) i.e. $y - y_1 = m_T(x - \frac{1}{4})$ with their tangent gradient and their y_1 or uses $y = mx + c$ with $(\frac{1}{4}$, their $y_1)$ and their tangent gradient.
	A1: $y = -6x + 3$ or $y = 3 - 6x$ or $a = -6$ and $b = 3$
(c)	**B1:** For the value 2/3 **not** 2/3 x **not** -3/2
	M1: Sets their gradient function dy/dx = their numerical gradient
	A1: Obtains $x = 9$
	dM1: Substitutes their x (from gradient equation) into original equation of curve C i.e. original expression $y =$
	A1: (9, -1) or $x = 9$, $y = -1$, or just $y = -1$
	Special Cases: In (b) Finds normal could get B1 M1 M0 A0 i.e. max of 2/4
	In (c) Uses **perpendicular** instead of **parallel** then award B0 M1 A0 M1 A0 i.e max 2/5 – see over

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19.

Question Number	Scheme	Marks
4. (a)	$y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3$	
	$\left\{\frac{dy}{dx} =\right\}\ 5(3)x^2 - 6\left(\frac{4}{3}\right)x^{\frac{1}{3}} + 2$	M1
	$= 15x^2 - 8x^{\frac{1}{3}} + 2$	A1 A1 A1
		[4]
(b)	$\left\{\frac{d^2y}{dx^2} =\right\}\ 30x - \frac{8}{3}x^{-\frac{2}{3}}$	M1 A1
		[2]
		6

	Notes	
(a)	**M1**: for an attempt to differentiate $x^n \to x^{n-1}$ to one of the first three terms of $y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3$. So seeing either $5x^3 \to \pm\lambda x^2$ or $-6x^{\frac{4}{3}} \to \pm\mu x^{\frac{1}{3}}$ or $2x \to 2$ is M1. **1st A1**: for $15x^2$ only. **2nd A1**: for $-8x^{\frac{1}{3}}$ or $-8\sqrt[3]{x}$ only. **3rd A1**: for $+2$ ($+c$ included in part (a) loses this mark). **Note:** $2x^0$ is A0 unless simplified to 2.	
(b)	**M1**: For differentiating $\frac{dy}{dx}$ again to give **either** • a correct follow through differentiation of their x^2 term • **or** for $\pm\alpha x^{\frac{1}{3}} \to \pm\beta x^{-\frac{2}{3}}$. **A1**: for any ***correct*** expression ***on the same line*** (accept un-simplified coefficients). For powers: $30x^{2-1} - \frac{8}{3}x^{\frac{1}{3}-1}$ is A0, but writing powers as one term eg: $(15\times 2x) - \frac{8}{3}x^{-\frac{4}{6}}$ is ok for A1. **Note:** Candidates leaving their answers as $\left\{\frac{dy}{dx} =\right\} 15x^2 - \frac{24}{3}x^{\frac{1}{3}} + 2$ and $\left(\frac{d^2y}{dx^2} =\right) 30x - \frac{24}{9}x^{-\frac{2}{3}}$ are awarded M1A1A0A1 in part (a) and M1A1 in part (b). **Be careful:** $30x - \frac{8}{3}x^{-\frac{1}{3}}$ will be A0. **Note:** For an extra term appearing in part (b) on the same line, ie $30x - \frac{8}{3}x^{-\frac{2}{3}} + 2$ is M1A0 **Note:** If a candidate writes in part (a) $15x^2 - 8x^{\frac{1}{3}} + 2 + c$ and in part (b) $30x - \frac{8}{3}x^{-\frac{2}{3}} + c$ then award (a) M1A1A1A0 (b) M1A1	

20.

Question Number	Scheme	Marks
	$P(4,-1)$ lies on C where $\mathrm{f}'(x)=\frac{1}{2}x-\frac{6}{\sqrt{x}}+3,\ x>0$	
7. (a)	$\mathrm{f}'(4)=\frac{1}{2}(4)-\frac{6}{\sqrt{4}}+3\ ;=2$	M1; A1
	T: $y--1=2(x-4)$	dM1
	T: $y=2x-9$	A1
		[4]
(b)	$\mathrm{f}(x)=\frac{x^{1+1}}{2(2)}-\frac{6x^{-\frac{1}{2}+1}}{(\frac{1}{2})}+3x\ (+c)$ or equivalent.	M1 A1
	$\{\mathrm{f}(4)=-1\Rightarrow\}\ \frac{16}{4}-12(2)+3(4)+c=-1$	dM1
	$\{4-24+12+c=-1\quad\Rightarrow\ c=7\}$	
	So, $\{\mathrm{f}(x)=\}\ \frac{x^2}{2(2)}-\frac{6x^{\frac{1}{2}}}{(\frac{1}{2})}+3x+7$	A1 cso
	$\left\{\text{NB: }\ \mathrm{f}(x)=\frac{x^2}{4}-12\sqrt{x}+3x+7\right\}$	**[4]**
		8

	Notes	
(a)	**1st M1:** for clear attempt at $\mathrm{f}'(4)$. **1st A1:** for obtaining 2 from $\mathrm{f}'(4)$. **2nd dM1:** for $y--1=(\text{their } \mathrm{f}'(4))(x-4)$ or $\frac{y--1}{x-4}=(\text{their } \mathrm{f}'(4))$ or full method of $y=mx+c$, with $x=4$, $y=-1$ and their $\mathrm{f}'(4)$ to find a value for c. **Note:** this method mark is dependent on the first method mark being awarded. **2nd A1:** for $y=2x-9$ or $y=-9+2x$ **Note:** This work needs to be contained in part (a) only.	
(b)	**1st M1**: for a clear attempt to integrate $\mathrm{f}'(x)$ with at least one correct application of $x^n\to x^{n+1}$ on $\mathrm{f}'(x)=\frac{1}{2}x-\frac{6}{\sqrt{x}}+3$. So seeing either $\frac{1}{2}x\to\pm\lambda x^{1+1}$ or $-\frac{6}{\sqrt{x}}\to\pm\mu x^{-\frac{1}{2}+1}$ or $3\to 3x^{0+1}$ is M1. **1st A1:** for correct un-simplified coefficients and powers (or equivalent) with or without $+c$. **2nd dM1:** for use of $x=4$ and $y=-1$ in an integrated equation to form a linear equation in c equal to -1. ie: applying $\mathrm{f}(4)=-1$. This mark is dependent on the first method mark being awarded. **A1:** For $\{\mathrm{f}(x)=\}\ \frac{x^2}{2(2)}-\frac{6x^{\frac{1}{2}}}{(\frac{1}{2})}+3x+7$ stated on one line where coefficients can be un-simplified or simplified, but must contain one term powers. Note this mark is for **correct solution only.** **<u>Note: For a candidate attempting to find f(x) in part (a)</u>** If it is clear that they understand that they are finding $\mathrm{f}(x)$ in part (a); ie. by writing $\mathrm{f}(x)=\ldots$ or $y=\ldots$ then you can give credit for this working in part (b).	

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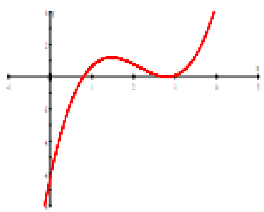
21.

Question	Scheme	Marks
1. **(a)**	$4x^3 + 3x^{-\frac{1}{2}}$	M1A1A1 (3)
(b)	$\frac{x^5}{5} + 4x^{\frac{3}{2}} + C$	M1A1A1 (3)
		6 marks
	Notes	
(a)	M1 for $x^n \to x^{n-1}$ i.e. x^3 or $x^{-\frac{1}{2}}$ seen 1st A1 for $4x^3$ <u>or</u> $6\times\frac{1}{2}\times x^{-\frac{1}{2}}$ (o.e.) (ignore any $+c$ for this mark) 2nd A1 for simplified terms i.e. <u>both</u> $4x^3$ <u>and</u> $3x^{-\frac{1}{2}}$ or $\frac{3}{\sqrt{x}}$ and no $+c$ $\left[\frac{3}{1}x^{-\frac{1}{2}} \text{ is A0}\right]$ Apply ISW here and award marks when first seen	
(b)	M1 for $x^n \to x^{n+1}$ applied to y only so x^5 or $x^{\frac{3}{2}}$ seen. Do not award for integrating their answer to part (a) 1st A1 for $\frac{x^5}{5}$ or $\frac{6x^{\frac{3}{2}}}{\frac{3}{2}}$ (or better). Allow $1/5x^5$ here but not for 2nd A1 2nd A1 for fully correct and simplified answer with $+C$. Allow $(1/5)x^5$ If $+C$ appears earlier but not on a line where 2nd A1 could be scored then A0	

22.

Question	Scheme	Marks
8. (a)	$[y = x^3 + 2x^2]$ so $\frac{dy}{dx} = 3x^2 + 4x$	M1A1 (2)
(b)		
	Shape 	B1
	Touching x-axis at origin	B1
	Through and not touching or stopping at -2 on x –axis. Ignore extra intersections.	B1 (3)
(c)	At $x = -2$: $\frac{dy}{dx} = 3(-2)^2 + 4(-2) = 4$	M1
	At $x = 0$: $\frac{dy}{dx} = 0$ (Both values correct)	A1 (2)
(d)		
	Horizontal translation (touches x-axis still)	M1
	$k - 2$ and k marked on positive x-axis	B1
	$k^2(2-k)$ (o.e) marked on negative y-axis	B1 (3)
		10 marks

	Notes
(a)	M1 for attempt to multiply out and then some attempt to differentiate $x^n \to x^{n-1}$ Do not award for $2x(x+2)$ or $2x(1+2)$ etc
Prod Rule	Award M1 for a correct attempt: 2 products with a + and at least one product correct A1 for both terms correct. (If $+c$ or extra term is included score A0)
(b)	1st B1 for correct shape (anywhere). Must have 2 clear turning points. 2nd B1 for graph touching at origin (not crossing or ending) 3rd B1 for graph passing through (not stopping or touching at) -2 on x axis and -2 marked on axis
SC	B0B0B1 for $y = x^3$ <u>or</u> cubic with straight line between $(-2,0)$ and $(0, 0)$
(c)	M1 for attempt at $y'(0)$ or $y'(-2)$. Follow through their 0 or – 2 and their $y'(x)$ <u>or</u> for a <u>correct</u> statement of zero gradient for an identified point on their curve that touches x-axis A1 for both correct answers
(d)	For the M1 in part (d) ignore any coordinates marked – just mark the shape. M1 for a horizontal translation of their (b). Should still touch x – axis if it did in (b) <u>Or</u> for a graph of correct shape with min. and intersection in correct order on +ve x-axis 1st B1 for k and k – 2 on the positive x-axis. Curve must pass through k – 2 and touch at k 2nd B1 for a correct intercept on negative y-axis in terms of k. Allow $(0, 2k^2 - k^3)$ (o.e.) seen in script if curve passes through –ve y-axis

23.

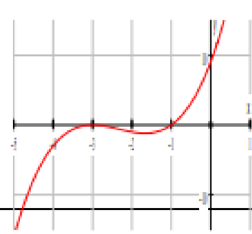
Question	Scheme	Marks
10. (a)	$\left(\frac{1}{2}, 0\right)$	B1 (1)
(b)	$\frac{dy}{dx} = x^{-2}$	M1A1
	At $x = \frac{1}{2}$, $\frac{dy}{dx} = \left(\frac{1}{2}\right)^{-2} = 4 \quad (= m)$	A1
	Gradient of normal $= -\frac{1}{m} \qquad \left(= -\frac{1}{4}\right)$	M1
	Equation of normal: $y - 0 = -\frac{1}{4}\left(x - \frac{1}{2}\right)$	M1
	$2x + 8y - 1 = 0 \qquad$ (*)	A1cso (6)
(c)	$2 - \frac{1}{x} = -\frac{1}{4}x + \frac{1}{8}$	M1
	$[= 2x^2 + 15x - 8 = 0]$ <u>or</u> $[8y^2 - 17y = 0]$	
	$(2x-1)(x+8) = 0$ leading to $x = \ldots$	M1
	$x = \left[\frac{1}{2}\right]$ or -8	A1
	$y = \frac{17}{8}$ (or exact equivalent)	A1ft (4)
		11 marks

	Notes	
(a)	B1 accept $x = \frac{1}{2}$ if evidence that $y = 0$ has been used. Can be written on graph. Use ISW	
(b)	1st M1 for kx^{-2} even if the '2' is not differentiated to zero. If no evidence of $\frac{dy}{dx}$ seen then 0/6 1st A1 for x^{-2} (o.e.) only 2nd A1 for using $x = 0.5$ to get $m = 4$ (correctly) (or $m = 1/0.25$) To score final A1cso must see at least one intermediate equation for the line after $m = 4$ 2nd M1 for using the perpendicular gradient rule on their m coming from their $\frac{dy}{dx}$ Their m must be a value not a letter. 3rd M1 for using a changed gradient (based on y') and their A to find equation of line 3rd A1cso for reaching printed answer with no incorrect working seen. Accept $2x + 8y = 1$ or equivalent equations with $\pm 2x$ and $\pm 8y$	
(c)	Trial and improvement requires sight of first equation. 1st M1 for attempt to form a suitable equation in one variable. Do not penalise poor use of brackets etc. 2nd M1 for simplifying their equation to a 3TQ and attempting to solve. May be $\Rightarrow$ by $x = -8$ 1st A1 for $x = -8$ (ignore a second value). If found y first allow ft for x if $x < 0$ 2nd A1ft for $y = \frac{17}{8}$ Follow through their x value in line or curve provided answer is > 0 This second A1 is dependent on <u>both</u> M marks	

24.

Question Number	Scheme	Marks
2. **(a)**	$\frac{dy}{dx} = 10x^4 - 3x^{-4}$ or $10x^4 - \frac{3}{x^4}$	M1 A1 A1 (3)
(b)	$\left(\int =\right) \frac{2x^6}{6} + 7x + \frac{x^{-2}}{-2} = \frac{x^6}{3} + 7x - \frac{x^{-2}}{2}$ $+ C$	M1 A1 A1 B1 (4) **7**
	Notes (a) M1: Attempt to differentiate $x^n \to x^{n-1}$ (for any of the 3 terms) i.e. ax^4 or ax^{-4}, where a is any non-zero constant or the 7 differentiated to give 0 is sufficient evidence for M1 1st A1: One correct (non-zero) term, possibly unsimplified. 2nd A1: Fully correct **simplified** answer. (b) M1: Attempt to integrate $x^n \to x^{n+1}$ (i.e. ax^6 or ax or ax^{-2}, where a is any non-zero constant). 1st A1: Two correct terms, possibly unsimplified. 2nd A1: All three terms correct and **simplified**. Allow correct equivalents to printed answer , e.g. $\frac{x^6}{3} + 7x - \frac{1}{2x^2}$ or $\frac{1}{3}x^6 + 7x - \frac{1}{2}x^{-2}$ Allow $\frac{1x^6}{3}$ or $7x^1$ B1: $+ C$ appearing at any stage in part (b) (independent of previous work)	

25.

Question Number	Scheme	Marks
10. **(a)**	 Shape (cubic in this orientation) **Touching** x-axis at **−3** **Crossing** at **−1** on x-axis Intersection at **9** on y-axis	B1 B1 B1 B1 (4)
(b)	$y = (x+1)(x^2 + 6x + 9) = x^3 + 7x^2 + 15x + 9$ or equiv. (possibly unsimplified) Differentiates their polynomial correctly – may be unsimplified $\frac{dy}{dx} = 3x^2 + 14x + 15$ **(*)**	B1 M1 A1 cso (3)

(c)	At $x=-5$: $\frac{dy}{dx} = 75-70+15=20$	B1
	At $x=-5$: $y=-16$	B1
	$y-("-16") = "20"(x-(-5))$ or $y =$ "$20x$" $+ c$ with (-5, -"16") used to find c	M1
	$y = 20x+84$	A1 (4)
(d)	Parallel: $3x^2+14x+15 = "20"$	M1
	$(3x-1)(x+5)=0 \quad x = \ldots$	M1
	$x = \frac{1}{3}$	A1
		(3) **14**
	Notes (a) Crossing at –3 is B0. Touching at –1 is B0 (b) M: This needs to be correct differentiation here A1: Fully correct simplified answer. (c) M: If the –5 and "-16" are the wrong way round or – omitted the M mark can still be given if a correct formula is seen, (e.g. $y-y_1 = m(x-x_1)$) otherwise M0. m should be numerical and not 0 or infinity and should not have involved negative reciprocal. (d) 1st M: Putting the derivative expression equal to their value for gradient 2nd M: Attempt to solve quadratic (see notes) This may be implied by correct answer.	

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26.

11. **(a)**	$\left(\frac{dy}{dx}=\right)\frac{3}{2}x^2 - \frac{27}{2}x^{\frac{1}{2}} - 8x^{-2}$	M1A1A1A1 (4)
(b)	$x=4 \Rightarrow y = \frac{1}{2}\times 64 - 9\times 2^3 + \frac{8}{4} + 30$	M1
	$= 32 - 72 + 2 + 30 \qquad = \underline{-8}$ *	A1cso (2)

(c)	$x=4 \quad \Rightarrow \quad y' = \frac{3}{2}\times 4^2 - \frac{27}{2}\times 2 - \frac{8}{16}$	M1
	$= 24 - 27 - \frac{1}{2} = \qquad -\frac{7}{2}$	A1
	Gradient of the normal = $-1 \div "\frac{7}{2}"$	M1
	Equation of normal: $y - -8 = \frac{2}{7}(x-4)$	M1A1ft
	$\underline{7y - 2x + 64 = 0}$	A1
		(6) **12**

	Notes	
(a)	1^{st} M1 for an attempt to differentiate $x^n \rightarrow x^{n-1}$ 1^{st} A1 for one correct term in x 2^{nd} A1 for 2 terms in x correct 3^{rd} A1 for all correct x terms. No 30 term and no $+c$.	
(b)	M1 for substituting $x = 4$ into $y =$ and attempting $4^{\frac{3}{2}}$ A1 note this is a printed answer	
(c)	1^{st} M1 Substitute x = 4 into y' (allow slips) A1 Obtains –3.5 or equivalent 2^{nd} M1 for correct use of the perpendicular gradient rule using their gradient. (May be slip doing the division) Their gradient must have come from y' 3^{rd} M1 for an attempt at equation of tangent or normal at P 2^{nd} A1ft for correct use of their changed gradient to find **normal** at P. Depends on 1^{st}, 2^{nd} and 3^{rd} Ms 3^{rd} A1 for any equivalent form with integer coefficients	

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27.

Question Number	Scheme	Marks
7.	$\frac{3x^2+2}{x} = 3x + 2x^{-1}$	M1 A1
	$(y' =) 24x^2, -2x^{-\frac{1}{2}}, +3 - 2x^{-2}$	M1 A1 A1A1
	$\left[24x^2 - 2x^{-\frac{1}{2}} + 3 - 2x^{-2}\right]$	
		6

	Notes
	1^{st} M1 for attempting to divide(one term correct) 1^{st} A1 for both terms correct on the same line, accept $3x^1$ for $3x$ or $\frac{2}{x}$ for $2x^{-1}$ These first two marks may be implied by a correct differentiation at the end. 2^{nd} M1 for an attempt to differentiate $x^n \to x^{n-1}$ for at least one term of their expression "Differentiating" $\frac{3x^2+2}{x}$ and getting $\frac{6x}{1}$ is M0 2^{nd} A1 for $24x^2$ only 3^{rd} A1 for $-2x^{-\frac{1}{2}}$ allow $\frac{-2}{\sqrt{x}}$. Must be simplified to this, not e.g. $\frac{-4}{2}x^{-\frac{1}{2}}$ 4^{th} A1 for $3-2x^{-2}$ allow $\frac{-2}{x^2}$. Both terms needed. Condone $3+(-2)x^{-2}$ If "+c" is included then they lose this final mark They do not need one line with all terms correct for full marks. Award marks when first seen in this question and apply ISW. Condone a mixed line of some differentiation and some division e.g. $24x^2-4x^{\frac{1}{2}}+3x+2x^{-1}$ can score 1^{st} M1A1 and 2^{nd} M1A1

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28.

Question number	Scheme	Marks
Q1	$x^4 \to kx^3$ or $x^{1/3} \to kx^{-2/3}$ or $3 \to 0$ (k a non-zero constant)	M1
	$\left(\frac{dy}{dx}=\right)$ $4x^3$.......... , with '3' differentiated to zero (or 'vanishing')	A1
	$\left(\frac{dy}{dx}=\right)$ .... $+\frac{1}{3}x^{-2/3}$ or equivalent, e.g. $\frac{1}{3\sqrt[3]{x^2}}$ or $\frac{1}{3(\sqrt[3]{x})^2}$	A1
		[3]
	1^{st} A1 requires $4x^3$, <u>and</u> 3 differentiated to zero. Having '+C' loses the 1^{st} A mark. Terms not added, but otherwise correct, e.g. $4x^3$, $\frac{1}{3}x^{-2/3}$ loses the 2^{nd} A mark.	

29.

Question number	Scheme	Marks
Q6	(a) $y = \dfrac{x^2 - 5x - 24}{x} = x - 5 - 24x^{-1}$ (or equiv., e.g. $x + 3 - 8 - \dfrac{24}{x}$)	M1 A1
	$\dfrac{dy}{dx} = 1 + 24x^{-2}$ or $\dfrac{dy}{dx} = 1 + \dfrac{24}{x^2}$	M1 A1 (4)
	(b) $x = 2$: $y = -15$ Allow if seen in part (a).	B1
	$\left(\dfrac{dy}{dx} =\right)$ $1 + \dfrac{24}{4} = 7$ Follow-through from candidate's non-constant $\dfrac{dy}{dx}$. This must be simplified to a "single value".	B1ft
	$y + 15 = 7(x - 2)$ (or equiv., e.g. $y = 7x - 29$) Allow $\dfrac{y+15}{x-2} = 7$	M1 A1 (4) **[8]**
	(a) 1st M: Mult. out to get $x^2 + bx + c$, $b \neq 0$, $c \neq 0$ and dividing by x (not x^2). Obtaining one correct term, e.g. x…….. is sufficient evidence of a division attempt. 2nd M: Dependent on the 1st M: Evidence of $x^n \to kx^{n-1}$ for one x term (i.e. not just the constant term) is sufficient). Note that mark is not given if, for example, the numerator and denominator are differentiated separately. A mistake in the 'middle term', e.g. $x + 5 - 24x^{-1}$, does not invalidate the 2nd A mark, so M1 A0 M1 A1 is possible. (b) B1ft: For evaluation, using $x = 2$, of their $\dfrac{dy}{dx}$, even if unlabelled or called y. M: For the equation, in any form, of a straight line through (2, '−15') with candidate's $\dfrac{dy}{dx}$ value as gradient. Alternative is to use (2, '−15') in $y = mx + c$ to find a value for c, in which case $y = 7x + c$ leading to $c = -29$ is sufficient for the A1). . (See general principles for straight line equations at the end of the scheme). Final A: 'Unsimplified' forms are acceptable, but… $y - (-15) = 7(x - 2)$ is A0 (unresolved 'minus minus').	

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30.

<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td rowspan="2">10. (a)</td><td>$\frac{1}{2}(9x+6x)4x$
or $2x \times 15x$
or $\left(\frac{1}{2}4x \times (9x-6x) + 6x \times 4x\right)$
or $6x^2 + 24x^2$
or $\left(9x \times 4x - \frac{1}{2}4x \times (9x-6x)\right)$
or $36x^2 - 6x^2$</td><td>M1: Correct attempt at the area of a trapezium.
Note that $30x^2$ on its own or $30x^2$ from incorrect work e.g. $5x \times 6x$ is M0.
If there is a clear intention to find the area of the trapezium correctly allow the M1 but the A1 can be withheld if there are any slips.</td><td rowspan="2">M1A1cso</td></tr>
<tr><td>$\Rightarrow 30x^2 y = 9600 \Rightarrow y = \frac{9600}{30x^2} \Rightarrow y = \frac{320}{x^2}$ *</td><td>A1: Correct proof with at least one intermediate step and no errors seen.
“ $y =$” is required.</td></tr>
<tr><td></td><td colspan="2"></td><td>[2]</td></tr>
<tr><td rowspan="7">(b)</td><td colspan="2">$(S =)\frac{1}{2}(9x+6x)4x + \frac{1}{2}(9x+6x)4x + 6xy + 9xy + 5xy + 4xy$</td><td>M1A1</td></tr>
<tr><td colspan="2">M1: An attempt to find the area of six faces of the prism. The 2 trapezia may be combined as $(9x+6x)4x$ or $60x^2$ and the 4 other faces may be combined as $24xy$ but all six faces must be included. There must be attempt at the areas of two trapezia that are dimensionally correct.
A1: Correct expression in any form.
Allow just $(S =)\ 60x^2 + 24xy$ for M1A1</td><td></td></tr>
<tr><td colspan="2">$y = \frac{320}{x^2} \Rightarrow (S =)\ 30x^2 + 30x^2 + 24x\left(\frac{320}{x^2}\right)$</td><td>M1</td></tr>
<tr><td colspan="2">Substitutes $y = \frac{320}{x^2}$ into their expression for S (may be done earlier). S should have at least one x^2 term and one xy term but there may be other terms which may be dimensionally incorrect.</td><td></td></tr>
<tr><td>So, $(S =)\ 60x^2 + \frac{7680}{x}$ *</td><td>Correct solution only.
“$S =$“ is not required here.</td><td>A1* cso</td></tr>
<tr><td colspan="2"></td><td>[4]</td></tr>
</table>

10(c)	$\frac{dS}{dx} = 120x - 7680x^{-2}$ $\left\{= 120x - \frac{7680}{x^2}\right\}$	M1: Either $60x^2 \to 120x$ or $\frac{7680}{x} \to \frac{\pm\lambda}{x^2}$	M1
		A1: Correct differentiation (need not be simplified).	A1 aef
	$120x - \frac{7680}{x^2} = 0$ $\Rightarrow x^3 = \frac{7680}{120}; = 64 \Rightarrow x = 4$	M1: $S' = 0$ and "their $x^3 = \pm$ value" or "their $x^{-3} = \pm$ value" Setting their $\frac{dS}{dx} = 0$ and "candidate's ft ***correct*** power of x = a value". **The power of x must be consistent with their differentiation**. If inequalities are used this mark cannot be gained until candidate states value of x or S from their x without inequalities. $S' = 0$ can be implied by $120x = \frac{7680}{x^2}$. Some may spot that $x = 4$ gives $S' = 0$ and provided they clearly show $S'(4) = 0$ allow this mark as long as S' is correct. (If S' is incorrect this method is allowed if their derivative is clearly zero for their value of x)	M1A1**cso**
		A1: $x = 4$ **only** ($x^3 = 64 \Rightarrow x = \pm 4$ scores A0) Note that the value of x is not explicitly required so the use of $x = \sqrt[3]{64}$ to give $S = 2880$ would imply this mark.	
	Note some candidates stop here and do not go on to find *S* – maximum mark is 4/6		
	$\{x = 4,\}$ $S = 60(4)^2 + \frac{7680}{4} = 2880$ (cm^2)	Substitute candidate's value of x ($\neq 0$) into a formula for S. **Dependent on both previous M marks.**	**dd**M1
		2880 **cso (Must come from correct work)**	A1 **cao and cso**
			[6]

10(d)	$\frac{d^2S}{dx^2} = 120 + \frac{15360}{x^3} > 0$ $\Rightarrow$ Minimum	M1: Attempt S'' $(x^n \to x^{n-1})$ **and** considers sign. This mark requires an attempt at the second derivative and **some consideration of its sign**. There does not necessarily need to be any substitution. An attempt to solve $S'' = 0$ is M0	M1A1ft
		A1: $120 + \frac{15360}{x^3}$ and > 0 **and** conclusion. Requires a **<u>correct</u>** second derivative of $120 + \frac{15360}{x^3}$ (need not be simplified) **<u>and</u>** a valid reason (e.g. > 0), **<u>and</u>** conclusion. Only follow through a correct second derivative i.e. x may be incorrect **but must be positive** and/or S'' may have been <u>evaluated</u> incorrectly.	
	A correct S'' followed by $S''("4") = "360"$ therefore minimum would score no marks in (d) **A correct S'' followed by $S''("4") = "360"$ which is positive therefore minimum would score both marks**		
			[2]
	Note parts (c) and (d) can be marked together.		
			Total 14

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31.

Question Number	Scheme	Marks
9. (a)	$\left\{\frac{dy}{dx}=\right\}\ 2x-16x^{-\frac{1}{2}}$	M1 A1
	$2x-16x^{-\frac{1}{2}}=0 \Rightarrow\ x^{\frac{3}{2}}=,\ x^{-\frac{3}{2}}=$,or $2x-=16x^{-\frac{1}{2}}$ then squared then obtain $x^3=$ [or $2x-16x^{-\frac{1}{2}}=0\ \Rightarrow x=4$ (no wrong work seen)]	M1
	$(x^{\frac{3}{2}}=8\ \Rightarrow)x=\ 4$	A1
	$x=4,\ y=\ 4^2-32\sqrt{4}+20=-28$ (ignore $y=100$ as second answer)	M1 A1
		(6)
(b)	$\left\{\frac{d^2y}{dx^2}=\right\}\ 2+8x^{-\frac{3}{2}}$	M1 A1
	$(\frac{d^2y}{dx^2}>0\ \Rightarrow)\ y$ is a minimum (there should be no wrong reasoning)	A1
		(3) **[9]**
(b)	**Alternative Method: Gradient Test:** M1 for finding the gradient either side of their x-value from part (a). A1 for both gradients calculated correctly to 1 significant figure, then using < 0 and > 0 respectively *maybe by use of sketch or table*. (See appendix for gradient values. This is **not ft their x**) A1 states minimum needs M1A1 to have been awarded.	

	Notes for Question 9
(a)	1[st] M1: At least one term differentiated correctly, so $x^2\to 2x$, or $32\sqrt{x}\to 16x^{-\frac{1}{2}}$, or $20\to 0$ A1: This answer or equivalent e.g. $2x-\frac{16}{\sqrt{x}}$ 2[nd] M1: Sets their $\frac{dy}{dx}$ to 0, and solves to give $x^{\frac{3}{2}}=,\ x^{-\frac{3}{2}}=$ *or* $x^3=$ after correct squaring or spots $x=4$ (NB $\left\{\frac{d^2y}{dx^2}=0\right\}$ *so* $2+8x^{-\frac{3}{2}}=0$ **is M0**) N.B. Common error: Putting derivative = 0 and merely obtaining $x=0$ is M0A0, then M0A0 for next two marks. (The first two marks in (a) and marks for second derivative may be earned in part (b).) A1: $x=4$ cao [$x=-4$ is A0 and $x=\pm 4$ is also A0] 3[rd] M1: Substitutes **their positive** found x **(NOT zero)** into $y=x^2-32\sqrt{x}+20,\ \ x>0$. **Should follow attempting to set $\frac{dy}{dx}=0$ and not setting** $\frac{d^2y}{dx^2}=0$ A1: -28 cao (Does not need to be written as coordinates)
(b)	M1: Attempts differentiation of their first derivative with at least one term differentiated correctly. Should be seen or referred to (in part (b)) in determining the nature of the stationary point. A1: Answer in scheme or equivalent A1: States minimum (Second derivative should be correct- can follow incorrect positive x. Needs M1A1 to have been awarded- should not follow incorrect reasoning – (need not say $\frac{d^2y}{dx^2}>0$ but should not have said $\frac{d^2y}{dx^2}=0$ for example)

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32.

Question Number	Scheme		Marks
8.	$y = 6 - 3x - \frac{4}{x^3}$		
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = -3 + \frac{12}{x^4}$ *or* $-3 + 12x^{-4}$	M1: $x^n \to x^{n-1}$ ($x^1 \to x^0$ or $x^{-3} \to x^{-4}$ or $6 \to 0$) A1: Correct derivative	M1 A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow -3 + \frac{12}{x^4} = 0 \Rightarrow x = ...$ **or** $\frac{\mathrm{d}y}{\mathrm{d}x} = -3 + \frac{12}{\sqrt{2}^4}$	$y' = 0$ and attempt to solve for x May be implied by $\frac{\mathrm{d}y}{\mathrm{d}x} = -3 + \frac{12}{x^4} = 0 \Rightarrow \frac{12}{x^4} = 3 \Rightarrow x = ...$ **or** Substitutes $x = \sqrt{2}$ into their y'	M1
	So $x^4 = 4$ and $x = \sqrt{2}$ **or** $\frac{\mathrm{d}y}{\mathrm{d}x} = -3 + \frac{12}{(\sqrt{2})^4}$ or $-3 + 12(\sqrt{2})^{-4} = 0$	Correct completion to printed answer with no errors by solving their $y' = 0$ or substituting $x = \sqrt{2}$ into their y'	A1
	For solving, allow e.g. $x^{-4} = \frac{1}{4} \Rightarrow x = \left(\frac{1}{4}\right)^{-\frac{1}{4}} = \sqrt{2}$ The minimum for verification is as in the scheme which could be implied by -3 + 3 = 0		
	Do not allow $x^4 = 4 \Rightarrow x = 1.41... = \sqrt{2}$ for the final A1		**(4)**
(b)	$x = -\sqrt{2}$	Awrt -1.41	B1
			(1)
(c)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{-48}{x^5}$ or $-48x^{-5}$	Follow through their first derivative from part (a)	B1ft
			(1)
(d)	An appreciation that either $y'' > 0 \Rightarrow$ a minimum or $y'' < 0 \Rightarrow$ a maximum	A generous mark that is independent of any previous work	B1
	Maximum at P as $y'' < 0$	Cso	B1
	Need a fully correct solution for this mark. y'' need not be evaluated but must be correct and there must be reference to P or to $\sqrt{2}$ and negative or < 0 and maximum. There must be no incorrect or contradictory statements (NB allow y'' = awrt-8 or -9)		
	Minimum at Q as $y'' > 0$	Cso	B1
	Need a fully correct solution for this mark. y'' need not be evaluated but must be correct **and part (b) must be correct** and there must be reference to P or to $-\sqrt{2}$ and positive or > 0 and minimum. There must be no incorrect or contradictory statements (NB allow y'' = awrt 8 or 9)		
			(3)
			[9]
	Other methods for identifying the nature of the turning points are acceptable. The first B1 is for finding values of *y* or d*y*/d*x* either side of $\sqrt{2}$ or their *x* at Q and the second and third B1's for fully correct solutions to identify the maximum/minimum.		

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33.

Question number	Scheme	Marks
8 (a)	$(h=)\dfrac{60}{\pi x^2}$ or **equivalent exact** (not decimal) expression e.g. $(h=)60\div\pi x^2$	B1 (1)
(b)	$(A=)2\pi x^2+2\pi xh$ or $(A=)2\pi r^2+2\pi rh$ or $(A=)2\pi r^2+\pi dh$ may not be simplified and may appear on separate lines	B1
	Either $(A)=2\pi x^2+2\pi x\left(\dfrac{60}{\pi x^2}\right)$ or As $\pi xh=\dfrac{60}{x}$ then $(A=)2\pi x^2+2\left(\dfrac{60}{x}\right)$	M1
	$A=2\pi x^2+\left(\dfrac{120}{x}\right)$ *	A1 cso (3)
(c)	$\left(\dfrac{dA}{dx}\right)=4\pi x-\dfrac{120}{x^2}$ or $=4\pi x-120x^{-2}$	M1 A1
	$4\pi x-\dfrac{120}{x^2}=0$ implies $x^3=$ (Use of >0 or <0 is M0 then M0A0)	M1
	$x=\sqrt[3]{\dfrac{120}{4\pi}}$ or answers which round to 2.12 (-2.12 is A0)	dM1 A1 (5)
(d)	$A=2\pi(2.12)^2+\dfrac{120}{2.12}$, $=85$ (only ft $x=2$ or 2.1 – both give 85)	M1, A1 (2)
(e)	**Either** $\dfrac{d^2A}{dx^2}=4\pi+\dfrac{240}{x^3}$ and sign considered (May appear in (c)) which is >0 and therefore minimum (most substitute 2.12 but it is not essential to see a substitution) (may appear in (c)) **Or** *(method 2)* considers gradient to left and right of their 2.12 (e.g at 2 and 2.5) **Or** (*method 3*) considers value of A either side Finds numerical values for gradients and observes gradients go from negative to zero to positive so concludes minimum **OR** finds numerical values of A, observing greater than minimum value and draws conclusion	M1 A1 (2)
		13 marks
Notes	(a) **B1**: This expression must be correct and in part (a) $\dfrac{60}{\pi r^2}$ is B0 (b) **B1:** Accept any equivalent correct form – may be on two or more lines. **M1 :** substitute their expression for h in terms of x into Area formula **of the form** kx^2+cxh **A1**: There should have been no errors in part (b) in obtaining this printed answer (c) **M1:** At least one power of x decreased by 1 **A1** accept any equivalent correct answer **M1:** Setting $\dfrac{dA}{dx}=0$ and finding a value for x^3 ($x^3=$ may be implied by answer). Allow $\dfrac{dy}{dx}=0$ **dM1:** Using **cube root** to find x **A1 :** For any equivalent correct answer (need 3sf or more) Correct answer implies previous M mark (d) **M1 :** Substitute the (+ve) x value found in (c) into equation for A and evaluate . **A1** is for **85 only** (e) **M1**: Complete method, usually one of the three listed in the scheme. For first method $A''(x)$ must be attempted and sign considered **A1**: Clear statements and conclusion. (numerical substitution of x is not necessary in first method shown, and x or calculation could be wrong but $A''(x)$ **must be correct** . Must **not** see 85 substituted)	

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34.

Question Number	Scheme		Marks
10.			
(a)	$V = 4x(5-x)^2 = 4x(25-10x+x^2)$		
	So, $V = 100x - 40x^2 + 4x^3$	$\pm\alpha x \pm \beta x^2 \pm \gamma x^3$, where $\alpha, \beta, \gamma \neq 0$	M1
		$V = 100x - 40x^2 + 4x^3$	A1
	$\frac{dV}{dx} = 100 - 80x + 12x^2$	At least two of their expanded terms differentiated correctly.	M1
		$100 - 80x + 12x^2$	A1 cao
			(4)
(b)	$100 - 80x + 12x^2 = 0$	Sets their $\frac{dV}{dx}$ from part (a) $= 0$	M1
	$\{\Rightarrow 4(3x^2 - 20x + 25) = 0 \Rightarrow 4(3x-5)(x-5) = 0\}$		
	$\{\text{As } 0 < x < 5\}\ x = \frac{5}{3}$	$x = \frac{5}{3}$ or $x =$ awrt 1.67	A1
	$x = \frac{5}{3},\ V = 4(\frac{5}{3})(5 - \frac{5}{3})^2$	Substitute candidate's value of x **where** $0 < x < 5$ into a formula for V.	dM1
	So, $V = \frac{2000}{27} = 74\frac{2}{27} = 74.074...$	Either $\frac{2000}{27}$ or $74\frac{2}{27}$ or awrt 74.1	A1
			(4)
(c)	$\frac{d^2V}{dx^2} = -80 + 24x$	Differentiates their $\frac{dV}{dx}$ **correctly** to give $\frac{d^2V}{dx^2}$.	M1
	When $x = \frac{5}{3}$, $\frac{d^2V}{dx^2} = -80 + 24\left(\frac{5}{3}\right)$		
	$\frac{d^2V}{dx^2} = -40 < 0 \Rightarrow V$ is a maximum	$\frac{d^2V}{dx^2} = -40$ and < 0 or negative and maximum.	A1 cso
			(2) **[10]**

	Notes
(a)	1^{st} M1 for a three term cubic in the form $\pm\alpha x \pm \beta x^2 \pm \gamma x^3$. Note that an un-combined $\pm\alpha x \pm \lambda x^2 \pm \mu x^2 \pm \gamma x^3$, $\alpha, \lambda, \mu, \gamma \neq 0$ is fine for the 1^{st} M1. 1^{st} A1 for either $100x - 40x^2 + 4x^3$ or $100x - 20x^2 - 20x^2 + 4x^3$. 2^{nd} M1 for any two of their expanded terms differentiated correctly. NB: If expanded expression is divided by a constant, then the 2^{nd} M1 can be awarded for at least two terms are correct. Note for un-combined $\pm\lambda x^2 \pm \mu x^2$, $\pm 2\lambda x \pm 2\mu x$ counts as one term differentiated correctly. 2^{nd} A1 for $100 - 80x + 12x^2$, **cao.** **Note:** See appendix for those candidates who apply the product rule of differentiation.

Question Number	Scheme	Marks
(b)	**Note you can mark parts (b) and (c) together.** Ignore the extra solution of $x = 5$ (and $V = 0$). Any extra solutions for V inside found for values inside the range of x, then award the final A0.	
(c)	M1 is for their $\frac{dV}{dx}$ differentiated **correctly (follow through)** to give $\frac{d^2V}{dx^2}$. A1 for all three of $\underline{\frac{d^2V}{dx^2} = -40}$ and $\underline{< 0 \text{ or negative}}$ and $\underline{\text{maximum}}$. Ignore any second derivative testing on $x = 5$ for the final accuracy mark. **Alternative Method: Gradient Test:** M1 for finding the gradient either side of their x-value from part (b) where $0 < x < 5$. A1 for <u>both gradients calculated correctly to the near integer, using > 0 and < 0 respectively or a correct sketch</u> and <u>maximum</u>. (See appendix for gradient values.)	

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35.

Question Number	Scheme	Marks
3	(a) $\left(\frac{dy}{dx} =\right)$ $2x - \frac{1}{2}kx^{-\frac{1}{2}}$ (Having an extra term, e.g. $+C$, is A0)	M1 A1 (2)
	(b) Substituting $x = 4$ into their $\frac{dy}{dx}$ and 'compare with zero' (The mark is allowed for : $<, >, =, \leq, \geq$) $8 - \frac{k}{4} < 0$ $\quad k > 32$ (or $32 < k$) <u>Correct inequality needed</u>	M1 A1 (2) **4**
	(a) M: $x^2 \to cx$ or $k\sqrt{x} \to cx^{-\frac{1}{2}}$ (c constant, $c \neq 0$) (b) Substitution of $x = 4$ into y scores M0. However, $\frac{dy}{dx}$ is sometimes <u>called</u> y, and in this case the M mark can be given. $\frac{dy}{dx} = 0$ may be 'implied' for M1, when, for example, a value of k or an inequality solution for k is found. <u>Working</u> must be seen to justify marks in (b), i.e. $k > 32$ alone is M0 A0.	

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36.

Question Number	Scheme	Marks
Q9 (a)	$\left[y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}} - 10 \right]$	
	$[y' =] \quad 6x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$	M1 A1
	Puts their $\frac{6}{x^{\frac{1}{2}}} - \frac{3}{2}x^{\frac{1}{2}} = 0$	M1
	So $x =$ $, \frac{12}{3} = 4$ (If $x = 0$ appears also as solution then lose A1)	M1, A1
	$x = 4, \quad \Rightarrow \quad y = 12 \times 2 - 4^{\frac{3}{2}} - 10, \quad$ so $y = 6$	dM1,A1 (7)
(b)	$y'' = -3x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{1}{2}}$	M1A1 (2)
(c)	[Since $x > 0$] It is a maximum	B1 (1) [10]

(a)	1st M1 for an attempt to differentiate a fractional power $x^n \to x^{n-1}$ A1 a.e.f – can be unsimplified 2nd M1 for forming a suitable equation using their $y'=0$ 3rd M1 for correct processing of fractional powers leading to $x = \ldots$ (Can be implied by $x = 4$) A1 is for $x = 4$ only. If $x = 0$ also seen and not discarded they lose this mark only. 4th M1 for substituting their value of x back into y to find y value. Dependent on three previous M marks. Must see evidence of the substitution with attempt at fractional powers to give M1A0, but $y = 6$ can imply M1A1
(b)	M1 for differentiating their y' again A1 should be simplified
(c)	B1 . Clear conclusion needed and must follow correct y'' It is dependent on previous A mark (Do not need to have found x earlier). (Treat parts (a),(b) and (c) together for award of marks)